## Menu of Instruction

### Daily: Classroom Environment of Numeracy
A classroom where students are surrounded by math: real-life math tasks, data analysis, math word walls, instruments of measurement, mathematical communication, class-created math charts, graphic organizers, calendars and evidence of problem solving.

### Daily: Calendar Math/Morning Work
This daily appetizer prepares the palate for the following entrees with Math Stretches, calendar activities, problems of the day, data work, incredible equations, review of skills to be maintained, and preview of skills to come.

### Your Choice: Whole Class Instruction
An excellent teaching strategy when students are working at the same level of achievement, to introduce lessons with an activating strategy, for teacher modeling and think-alouds, for read alouds of math-related literature, to review previously mastered skills, as preparation for work in cooperative groups, or for paper and pencil assessments.

### Your Choice: Small Group Instruction
Students are instructed in small groups whose composition changes based on their needs. The individualized preparation for these groups offers tantalizing opportunities to introduce new concepts, practice new skills, work with manipulatives, provide intensive and targeted instruction to struggling learners, introduce activities that will later become part of Math Workshop, conduct informal assessments, and re-teach based on student needs.

### Your Choice: Math Workshop
Independent work by students either individually, in pairs, or in cooperative groups. The work may be follow-up from whole class or small group instruction, practice of previously mastered skills, investigations, math games, math journals, or interdisciplinary work.

### Daily: Conferencing
To enhance learning, teachers confer individually with students, informally assessing understanding, providing opportunities for one on one mathematical communication, and determining teaching points for individual students as well as for the class.

### Daily: Assessment
A generous helping of Assessment for Learning to inform instruction with a dollop of Assessment of Learning to top off each unit.

Yahoo Guided Math Group: http://tech.groups.yahoo.com/group/guidedmath/
Monday: What does the problem ask me to find?

Tuesday: What information does the problem give me?

Wednesday: What strategy or operation will I use to solve this problem?

Thursday: This is how I solved the problem.

Friday: I wrote my solution in a complete sentence and checked to be sure it makes sense.
Using Math Stretches to Promote Mathematical Literacy (cont.)

Planning Math Stretches (cont.)

The Math Stretches provided in this book offer examples of activities teachers can use to stimulate students’ mathematical thinking as the day begins. Teachers are not limited to the activities included here. They may create engaging stretches specifically designed to fit the needs of their classes. When creating Math Stretches, it is important to keep in mind their characteristics:

- They are very brief.
- They can be completed by students independently.
- They prompt students to think mathematically.
- They encourage students to make mathematical connections to themselves, to other math concepts, and/or to math in other content areas.
- They generate mathematical communication.
- They provide opportunities to clarify understanding of mathematical vocabulary.
- They offer students repeated chances to revisit important mathematical big ideas.

Some teachers find it convenient to repeatedly use the same five stretches each week, assigning one regularly for each day of the week (Figure 2). Students learn the schedule and know what to expect as they enter the classroom. An example of a week’s worth of morning Math Stretches for a third grade class is shown in Figure 2 below.

<table>
<thead>
<tr>
<th>Day of the Week</th>
<th>Morning Stretch</th>
<th>Topic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monday</td>
<td>Number of the Day</td>
<td>180</td>
</tr>
<tr>
<td>Tuesday</td>
<td>What’s Next?</td>
<td>1, 3, 9, 27, …</td>
</tr>
<tr>
<td>Wednesday</td>
<td>How Did My Family Use Math Last Night?</td>
<td>Real-life mathematical connections</td>
</tr>
<tr>
<td>Thursday</td>
<td>A _____ Makes Me Think Of…</td>
<td>Multiplication</td>
</tr>
<tr>
<td>Friday</td>
<td>Data Collection</td>
<td>Where would you rather go on a field trip?</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• science museum</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• planetarium</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• aquarium</td>
</tr>
</tbody>
</table>

Math Huddles to Discuss the Math Stretch (cont.)

As students become more proficient at sharing their mathematical ideas related to the Math Stretch, teachers step back to become facilitators of the Math Huddle. Broad, open-ended questions posed by the teacher may start the discussion or move it along if it stalls. Other than that, the teacher ensures that the talk remains focused and that all students feel comfortable participating.

Eventually, when students have demonstrated their understanding of the process, they are asked to record their own analysis in their Math Journals prior to the class huddle. Writing ideas in their journals prior to the discussion helps students to formalize their ideas so that they can actually see, consider, and revise them if needed (O’Connell 2007, 134). This in no way eliminates the need for a discussion, however.
How Many Ways Can We Represent This Number? Stretch

Grades 3-5

- Understands equivalent forms of basic percents, fractions, and decimals (e.g., 1/2 is equivalent to 50% is equivalent to .5) and when one form of a number might be more useful than another
- Uses models (e.g., number lines, two-dimensional and three-dimensional regions) to identify, order, and compare numbers

Overview

With the How Many Ways Can We Represent This Number? Stretch, students are asked to use numbers, words, or drawings to represent the number the teacher displays on the IWB. The number students are asked to represent can vary depending on the class’s current academic focus and level of mastery.

Materials

- How Many Ways Can We Represent This Number? IWB file or a teacher created chart. The number that the teacher selects is written in the center of the chart in large numerals.
- An appropriate recording tool

Warming Up for the Stretch

The How Many Ways Can We Represent This Number? Stretch should be introduced during Calendar Board instruction. The teacher should model a variety of ways a number can be represented and challenge students to suggest additional representations. The concept of equal representations should be demonstrated with a number that can be converted easily into several forms of representation. Students need to understand the representation of a number may vary, but the value of each representation is the same.

Stretch Procedures

- The How Many Ways Can We Represent This Number? IWB or teacher made chart is displayed as students enter the classroom in the morning.
- Students will use the IWB pen or marking tool to write a number, number expression, number sentence, or create a picture that is an equal representation of the given number along with their initials.
- When all students have added a representation, the teacher calls the students together for a Math Huddle. The teacher will ask students to check the accuracy of the representations and, using math language, to explain how they determined what representations to add to the chart and to justify their representations.

Suggested Questions for Informal Assessment: Math Huddle

- Look carefully at the chart we have created. Are there any representations that you wonder about? Why do you wonder about that representation?
- What math reasoning did you use to determine that your representation was equal to the number on the chart? Does anyone agree or disagree with that reasoning? Why or why not?
- Why do you think it is important to be able to represent numbers in different ways? When do we usually use number words to represent numbers? When do we use numerals? When do we use
pictures or diagrams? When do we use number expressions or number sentences? Why do we sometimes choose one method of representation rather than another?

• How does understanding whole numbers, fractions, decimals, and percentages help you represent numbers in multiple ways? How does knowing about addition, subtraction, multiplication, and division help?

• When would we see a number and want to find another representation of it? How would this help you if you are shopping and see something on sale? How about when you are sharing a pizza? When can finding more than one way to represent a number or value be important in our daily lives?

**What It Looks Like: Stretch Snapshot**

The How Many Ways Can We Represent This Number? Stretch allows teachers to assess the ability of their students to create numeric and pictorial representation of numbers. As students display their representations, they are making connections between different kinds of numbers -- in this case whole numbers, fractions, decimals, and percents. As students determine their representations, it is important for the teacher to engage them in conversation about their mathematical reasoning.

This fifth grade class has completed a unit on percents. The teacher is using the Math Stretch to reinforce students’ conceptual understanding of equal representations of a percent. The young mathematicians are making connections between a percent, a decimal, a fraction, a diagram, and word equivalents. In the Stretch today, the given value is twenty percent.

As the teacher gathers students into the Math Huddle, she sees that Susan has drawn a fraction bar divided into six parts with one part shaded. While working on fractions, Susan struggled with conceptual understanding of what fractions represent. The teacher believes that when Susan was simplifying twenty-hundredths, she converted the percentage to a fraction and simplified it to 1/5 accurately, but then incorrectly drew her fraction bar using six as the base. The teacher wonders if Susan understood that the numerator indicates the number of sections to be shaded, but mistakenly thought that the denominator indicates the number of sections that should be unshaded in the fraction bar.

*Teacher: I see we have a fraction bar representation. Class, if you agree that this is an equal representation of twenty percent, thumbs up. If not, thumbs down. Some of us don’t think this represents twenty percent. Let’s check to see if it is correct. Susan, I see your initials by that representation. Into how many parts is the bar divided and how many of those parts are shaded?*

*Susan: I divided it into six parts and shaded one.*

*Teacher: Explain for us how you determined that it should be divided into sixths.*

*Susan: I knew that twenty percent was just twenty over one hundred as a fraction. Then I divided it by four since four goes into twenty evenly.*

The teacher writes the fraction twenty-hundredths. She places a division sign and the number four to the right of both the numerator and the denominator.

*Teacher: Okay, Class, let’s divide together. Twenty divided by four is what?*

*Steve: Five!*

Laney Sammons, Mathematics Consultant
Teacher: Excellent! Now, when we divide one hundred by four, what do we get?

Steve: That’s twenty-five! That was my number, five twenty-fifths.

Teacher: Oh, I see that one. Class, can we simplify that any further?

Susan: Yes, we can divide it by five over five. That would be one fifth. So, my fraction bar is right! One part is shaded, and five parts are not.

Teacher: Does everyone agree with Susan?

Steve: I don’t agree.

Teacher: Why not?

Steve: The denominator tells us how many equal parts a whole is divided into. So, the whole bar should be divided into five parts, not six.

Susan: Oh, I forgot. I need to fix my diagram.

Teacher: Very good, Susan. Now look at your diagram. Into how many equal parts should your bar be divided?

Susan: Five. I’m going to take one part off.

Teacher: You have shaded one part. How many parts should be unshaded?

Susan: Four parts should be unshaded, but I shaded the right amount.

Teacher: Very good! Why don’t you come up here and make that change! Class, when you draw a diagram, which part of the fraction tells us into how many equal parts we need to divide the bar?

Steve: The denominator.

Teacher: What does the numerator represent?

Susan: The number of parts that I shade.

With this Math Huddle, the teacher was able to diagnose Susan’s misconception and to correct it. The discussion provided a review of fractions and of simplifying fractions for the entire class, thus maintaining student understanding of a skill previously taught.
How Many Ways Can We Represent This Number? *stretch (cont.)*

Sample Chart

How many ways can we represent this number?

Use words, numbers, or pictures to represent this number. Add your initials to your representation.

\[
\begin{align*}
\text{SS} & : 15 \\
\text{100} & : \frac{1}{5} \\
\text{SP} & : \frac{1}{5} \\
\text{CM} & : \frac{1}{5} \\
\text{CT} & : 0.20 \\
\text{PM} & : \frac{4}{20} \\
\text{HW} & : 0.2 \\
\text{SR} & : \frac{5}{25}
\end{align*}
\]
### STRATEGIES FOR TEACHING A SMALL-GROUP LESSON

1. Briefly introduce the lesson by providing **supportive strategies** for learners. Use *modeling and think-alouds*.

2. Provide students with a **clear understanding** of the activity or task on which they will work including the criteria for success.

3. Encourage students to **try a variety of strategies** to solve the problem or complete the activity by creating a **supportive, risk-free learning environment**.

4. **Scaffold student learning** by giving just enough support to move students to the next level of understanding and proficiency.

5. Provide many opportunities for **rich mathematical discourse** both between pairs of students, between you and individual students, and among the group. Listen carefully to understand their mathematical thinking. Use **probing questions** that encourage students to **do most of the talking**.

6. Give students **specific, descriptive feedback** as they work and encourage them to self-assess their work based on the established **criteria for success**.
**MATH WORKSHOP ORGANIZATION**

<table>
<thead>
<tr>
<th>GUIDE</th>
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<tbody>
<tr>
<td><strong>G</strong> Games for Mathematicians</td>
</tr>
<tr>
<td><strong>U</strong> Using What We Know</td>
</tr>
<tr>
<td><strong>I</strong> Independent Math Practice</td>
</tr>
<tr>
<td><strong>D</strong> Developing Fluency</td>
</tr>
<tr>
<td><strong>E</strong> Expressing Mathematical Ideas</td>
</tr>
</tbody>
</table>

| Math games to maintain previously mastered math concepts and skills |
| Problem solving or other challenges in which students draw upon their mathematical understanding and skills |
| Materials used to teach previously mastered concepts incorporated into station tasks Paper and pencil tasks may be included. |
| Tasks that help students develop number sense and mental math skills |
| Focus on mathematical vocabulary and communication May include use of math journals or math vocabulary notebooks |

<table>
<thead>
<tr>
<th>Red Team</th>
<th>Blue Team</th>
<th>Yellow Team</th>
<th>Green Team</th>
<th>Orange Team</th>
</tr>
</thead>
</table>

**Other possible models:**
- Three groups: teacher, written task, math games
- 8 to 10 Math Stations: work in pairs, task assigned or by choice
- Regular stations: problem solving, journal, games, facts practice, computer, listening center, number writing, teacher, free choice
- Rotation: teacher, math games, math card game, individual work
- Stations for each math domain
## Top 10 Guidelines for Independent Math Stations

1. Start with academic goals in mind rather than materials at hand.

2. Use simple materials with simple procedures. Don’t work yourself to death.

3. Tasks do NOT have to be changed weekly. Change them when students are tired of them or they no longer meet the learning needs of students.

4. Plan tasks in which the content may vary, but the basic game or procedures remain the same.

5. Assign tasks to address skills and concepts that have already been taught--use as review and maintenance. Students must be able to complete them successfully independently.

6. Model and have students role play the tasks before assigning them to be done independently.

7. Provide some form of accountability. This might be a recording sheet or weekly checklist of tasks to be completed.

8. Provide Talking Point cards to accompany the tasks. Model how students should use these.

9. Have students work in pairs rather than larger groups. When more students are involved, there is often more time spent off task and more problems arise.

10. Have clear rituals and routines. Spend the first two weeks of school teaching these. Whenever necessary.
Conferences should be **brief** and conducted in a conversational tone with one student at a time.

**The Structure of a Math Conference**  
(Adapted from Lucy Calkins’ writing conference structure)

- **Research:** The teacher asks about a student’s work and listens for evidence that the child understands the concepts being taught. From this, she determines additional teaching points both for the individual student and the class as a whole.

- **Compliment:** The teacher gives an authentic compliment based on some aspect of the child’s work.

- **Teaching Point:** The teacher presents a teaching point emphasizing that this is something “really good mathematicians” do.

- **Link:** Finally, the conference ends with the teacher encouraging the students to always think about this idea and use it in similar situations.
Balanced Assessments for Grouping Students

1. Kinds of assessment (observation, communication, product)
2. Formative/summative
3. Formal/informal/self-assessments

When planning a lesson:
- Consider what are the basic foundational understandings and skills students must have to be successful with the lesson you will teach.
- Consider what would be evidence that students have already mastered the standards to be taught.
- Plan a simple assessment that will indicate if students have gaps in that foundational knowledge or if they have already achieved mastery.
- Group using this data. Plan ways to plug gaps or to provide additional challenges.
**Small-Group Lesson Planning Form: Grade 1 Model Lesson**

**Overview:**

M1N1. Students will estimate, model, compare, order, and represent whole numbers up to 100.
   a. Represent numbers up to 100 using a variety of models, diagrams, and number sentences.
      Represent numbers larger than 10 in terms of tens and ones using manipulatives and pictures.
M1N3. Students will add and subtract numbers less than 100, as well as understand and use the inverse relationship between addition and subtraction.
   Identify one more than, one less than, 10 more than, and 10 less than a given number.

Students are challenged to determine what the similarities are when presented with groups of ten (pennies, a dime, a row from a hundreds chart, a ten frame). They learn to use their knowledge of tens to determine ten more and ten less with a hundreds chart.

**Informal Assessment:** Have students draw twelve circles. Ask them to circle ten of them. Ask them to write how many they would have if they had ten more than that.

<table>
<thead>
<tr>
<th>Small-Group Lesson</th>
<th>Need for Additional Challenge</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Connection:</strong> Your teacher tells me that you have been working with groups of tens and groups of ones. We are going to talk more about that today. (Put out the groups of ten on the table.) Who can think of a way in which these are alike? How are they similar? (Lead students to see that each is a ten, just represented in different ways.) How else have you represented tens?</td>
<td>Challenge students to find ten more or ten less than a three digit number or find a hundred more or less.</td>
</tr>
<tr>
<td><strong>Teaching Point:</strong> Today we are going to learn how we can use what we know about tens and ones to find out what ten more than a number is or ten less using mental math. How do we show how many groups of tens we have when we write a number? How do we show how many ones we have?</td>
<td></td>
</tr>
<tr>
<td><strong>Active Engagement:</strong> Let’s look at a hundred chart. (Have students find ten more or ten less than a given number and explain how they find their answers.) Let’s play a game using what we know. (Play “I have____. Who has ____?”)</td>
<td></td>
</tr>
<tr>
<td><strong>Link:</strong> Whenever you are asked to find what ten more or ten less than a number is remember to use these strategies. Mathematicians use what they know to find solutions to problems.</td>
<td></td>
</tr>
<tr>
<td><strong>Reflection:</strong> What did you learn or think about mathematically today?</td>
<td></td>
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</tbody>
</table>

**Need for Rebuilding Foundational Knowledge**

(List Common Gaps and Ways to Address Them)

- Inability to accurately count out objects: Practice rote counting to be sure students have acquired that skill. If students can rote count, have students practice counting objects by touching each one and moving the objects counted to another area.
- Inability to circle groups of ten: Use concrete objects. Have students place ten objects on individual mats or cups. When they can do this, have them either use ink stamps to create a picture with ten or more stamped images and then circle groups of ten. Or use printed gift wrap and have students circle groups of ten.
# Small-Group Lesson Planning Form

<table>
<thead>
<tr>
<th>Overview:</th>
<th></th>
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</thead>
<tbody>
<tr>
<td>Informal Assessment:</td>
<td></td>
</tr>
<tr>
<td>Small-Group Lesson Connection: Need for Additional Challenge</td>
<td></td>
</tr>
<tr>
<td>Teaching Point:</td>
<td></td>
</tr>
<tr>
<td>Active Engagement: Need for Rebuilding Foundational Knowledge (List Common Gaps and Ways to Address Them)</td>
<td></td>
</tr>
<tr>
<td>Link:</td>
<td></td>
</tr>
</tbody>
</table>
Putting It Into Practice

Sample Guided Math Schedule

♦ **Math Warm-up** (15–20 minutes max). It *may* include any of these:
  - Calendar Board
  - Math Stretches
  - Daily Review
  - Problem of the Week

♦ **Mini Lesson** (10 minutes—*only* if needed)

♦ **Math Workshop with Small-Group Instruction** (50–60 minutes)
Meeting with Groups

<table>
<thead>
<tr>
<th>Group 1 (solid background knowledge)</th>
<th>10 minutes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Group 2 (average background knowledge)</td>
<td>15 minutes</td>
</tr>
<tr>
<td>Group 3 (significant gaps in background knowledge)</td>
<td>35 minutes</td>
</tr>
<tr>
<td>Total time:</td>
<td>60 minutes</td>
</tr>
</tbody>
</table>

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<tr>
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<th></th>
<th></th>
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<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Felisa, Nori, Brad, Ray, Davisha</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>2. Ricardo, Marcus, Karina, Tameshia, Portia</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3. Dimitri, Tonya, Mary, Beth, Carlos, Keon</td>
<td></td>
<td></td>
<td>X</td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>4. Min, Monica, Quin, Lucas, Rosa</td>
<td></td>
<td></td>
<td></td>
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<td>X</td>
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</table>