HOW TO USE MATH ERROR ANALYSIS TO IMPROVE INSTRUCTION

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Topics for Today

• Overall purpose is to improve student learning outcomes in mathematics through the use of more effective instructional techniques

• Conducting Error Analysis to inform Instruction
  – How to identify and recognize common error patterns in students’ math solutions to inform instruction
  – How and where to fit error analysis into your use of student performance data and your data decision-making process.
  – Reteaching based on an identified error pattern.
  – Using micro-instruction progressions to target specific error patterns
  – Capitalizing on the use of examples and non-examples when reteaching

• Conclusion and Wrap-Up

Components of Effective Mathematics Programs

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Learning Processes-NMAP-2008

• To prepare students for Algebra, the curriculum must simultaneously develop conceptual understanding, computational fluency, factual knowledge and problem solving skills.

• Limitations in the ability to keep many things in mind (working-memory) can hinder mathematics performance.
  - Practice can offset this through automatic recall, which results in less information to keep in mind and frees attention for new aspects of material at hand.
  - Learning is most effective when practice is combined with instruction on related concepts.
  - Conceptual understanding promotes transfer of learning to new problems and better long-term retention.

Instructional Practices-NMAP-2008

Research on students who are low achievers, have difficulties in mathematics, or have learning disabilities related to mathematics tells us that the effective practice includes:

✓ Explicit methods of instruction available on a regular basis
✓ Clear problem solving models
✓ Carefully orchestrated examples/sequences of examples.
✓ Concrete objects to understand abstract representations and notation.
✓ Participatory thinking aloud by students and teachers.
Learner Characteristics

• **Strategic Learners**
  – Able to analyze a problem and develop a plan
  – Able to organize multiple goals and switch flexibly from simple to more complicated goals
  – Access their background knowledge and apply it to novel tasks
  – Develop new organizational or procedural strategies as the task becomes more complex
  – Use effective self-regulated strategies while completing a task
  – Attribute high grades to their hard work and good study habits
  – Review the task-oriented-goals and determine whether they have been met


• **Non-Strategic Learners**
  – Unorganized, impulsive, unaware of where to begin an assignment
  – Unaware of possible steps to break the problem into a manageable task, possibly due to the magnitude of the task
  – Exhibit problems with memory
  – Unable to focus on a task
  – Lack persistence
  – Experience feelings of frustration, failure, or anxiety
  – Attribute failure to uncontrollable factors (e.g., luck, teacher's instructional style)

Essential Question for Teachers

• Essential Question
  — What did I do “instructionally different” to support learning for the struggling students?

• Asked during instructional planning and after instructional delivery!!!

Error Pattern Analyses

• As math problems become more complex, students need to go through a series of steps to solve problems.
• An error in any of these steps can cause failure in the final response.
• As a result, it is important to identify errors, especially error patterns, and provide targeted instruction to correct the error.
Classroom Instructional Assessment Cycle

- Generally includes 5 Components
  1. Develop learning goals and outcomes
  2. Plan and deliver Instruction
  3. Assess student performance
  4. Analyze results using multiple data sources
  5. Use data analysis results to inform instruction and/or reteaching

Error Analysis Process

Source: Ohio Improvement Process

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Error Analysis Purpose

Error analysis

- Involves reviewing the student’s independent work (e.g., seatwork, quizzes, tests, progress monitoring) to identify specific error types and patterns
- Helps to set priorities for teaching

Error patterns

- Indicate areas in need of further instruction
- Constitute a database for determining what content and strategies to teach
Error Analysis Process

1. Identify errors on student’s independent work
2. Categorize errors by type
   - Conceptual
   - Factual
   - Procedural
   - Careless
3. Look for error patterns within each error type and across each error type
4. Provide instruction to remediate the error

Mathematical Error Types

1. Conceptual Errors
2. Factual Errors
   - AKA Computational Errors
3. Procedural Errors
4. Careless Errors

Errors occurring more than two times are considered a pattern and will require targeted instruction to correct.
Conceptual Errors

- Conceptual Errors
  - Students have developed misconceptions and have very poor understanding of concepts, procedures, and appropriate application.
  - Conceptual errors could also be related to vocabulary.
  - Difficult to differentiate between conceptual errors and procedural errors
  - May require significant amounts of reteaching at the concrete and/or representational levels.

Types of Factual Errors

1. Sign Identification
2. Digit Identification
3. Counting errors
4. Computational Errors
5. Place Value Identification
6. Vocabulary Terminology
7. Incorrect formula use

Often possible to correct with minimal instructional time.
Types of Procedural Errors

- Equivalence
- Multiplying all Digits
- Regrouping
- Exponent Procedures
- Correct Alignment
- Equivalent fractions
- Decimal Position
- Solving simple equations

Requires specific and targeted remediation that matches the procedural error. These are most common type of error and will continue to occur across school years if not remediated.

**Practice is generally not effective for remediating procedural errors**

Getting Started with Identifying Error Patterns

- Analyze student’s work to determine a common mistake.
  - Review independent work from 3 different samples
- Some mistakes are “careless” but, many mistakes present a common error pattern.
- Very often, the student develops the error through a poorly applied algorithm, using at incorrect time, or mixing up of procedures.
  - Very evident in computation of fractions
- Error patterns in student work is more common than most teachers realize.
Case Study in Error Analysis

Tyler’s Error Analysis
1. Examine Student’s Independent Work
   – Basic Facts
   – Computation
     • Addition
     • Subtraction
     • Multiplication
     • Division
2. Identify specific error patterns (Error Analysis)
3. Develop a “Profile” of errors

Case Study in Error Analysis

Review across 3 Samples of Tyler’s Work

Steps:
1. Identify all problems that student answered incorrectly.
2. Record the problem and the student’s response.
3. Record incorrect problems on one data sheet.
4. Look across incorrect problems for patterns that may emerge.
5. Record your “hypothesis” of the error pattern and possible causes

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# Case Study in Error Analysis

Data Recording sheet of Tyler’s Basic Fact Errors

<table>
<thead>
<tr>
<th>FACTS</th>
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<th>Multiplication</th>
<th>Division</th>
</tr>
</thead>
<tbody>
<tr>
<td>Error Patterns</td>
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<td></td>
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</tbody>
</table>

## Error Patterns
- Off by 1
- No Errors
- Sign (+)
- Sign (-)

### Observed Errors
- Signs: (x,-)
  - Observed 6’s, 7’s, 8’s, 6’s, 7’s, 8’s
  - 9’s (>5’s) 9’s (>5’s)

### Examples
- **Addition**
  - 4+2=5
  - 10+3=12
  - 9+7=15
  - 9+5=45 (x)
- **Subtraction**
  - 3+3=0 (-)
  - 10+7=70 (x)
- **Multiplication**
  - 8x9
  - 6x9
  - 9x1=10 (+)
  - 7x8
  - 5x1=6 (+)
  - 6x12=NA
  - 3x9=12 (+)
- **Division**
  - 56 ÷ 8 = 7
  - 8 ÷ 4 = 2
  - 144 ÷ 12 = 12
  - 72 ÷ 9 = 8
  - 36 ÷ 6 = 6
  - 14 ÷ 7 = 2
  - 33 ÷ 11 = NA
  - 42 ÷ 6 = 7
  - 42 ÷ 7 = 6

## Student Profile

Record all errors observed in student work by identifying specific areas in need of "attention" that will better focus instruction.
Case Study in Error Analysis

Fluency and Automaticity Instructional Strategies

• Fluency of facts is vital, but instruction for conceptual understanding must occur first
• Fluency activities must be cumulative and **REGULAR**
  – Newly introduced facts receive intensive practice, while previously introduced facts receive less intensive, but still **SYSTEMATICALLY PLANNED**.
• Fluency building activities should **NOT** use up all of the allocated math time...**5-10 minutes**
• Fact **fluency instruction** is often **overlooked** by most math programs or provide ineffective practice opportunities
• Refer to fluency and automaticity strategies covered during in previous webinars

Procedural Errors: Addition

6 Separate Steps: (Procedural and Factual Errors)

1. Addition Facts: Are the single digit addition sums computed correctly?
2. Regroup (Inside): Are regroupings assigned to the proper column?
3. Regroup (Outside): Is the last regrouping part of the answer?
4. Adding the Regroup: Are regrouped numbers added with the proper column?
5. Decimal: Is the decimal in the correct place?
Procedural Errors: Subtraction

4 Separate Steps:

1. Direction of Subtraction: Is subtraction always carried out in the proper direction?
2. Subtraction Facts: Are the single digit subtraction tasks computed correctly?
3. Where to Regroup: Does the student regroup from the correct column?
4. Conversions: After regrouping, does the student make the appropriate conversions in the adjacent columns?

Case Study in Error Analysis

Review Tyler’s completed subtraction problem solutions and identify the error pattern in his work.

\[
\begin{array}{cccc}
1.1812 & 8.72 & 826 & 25.610 \\
-0.94 & -4.23 & -21 & -0.33 \\
1.98 & 4.59 & 805 & 25.37 \\
\end{array}
\]
Case Study in Error Analysis

Tyler’s error analysis profile can be used to guide and focus instruction

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<td></td>
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<td></td>
</tr>
<tr>
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<td>6x9</td>
<td></td>
<td>8+4=4 (-)</td>
<td></td>
</tr>
<tr>
<td>9+7=15</td>
<td>9x1=10 (+)</td>
<td></td>
<td>144+12=3</td>
<td></td>
</tr>
<tr>
<td>9+5=45 (x)</td>
<td>7x8</td>
<td>72+9</td>
<td>36+6</td>
<td></td>
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<td></td>
<td>42+7</td>
<td></td>
</tr>
</tbody>
</table>

Error Patterns
- Off by 1 Signs: (x,-) No Errors Observed
- Sign (+) 6's, 7's, 8's, 9's (>5's)
- Sign (-) 6's, 7's, 8's, 9's (>5's)

Subtraction Error Pattern:
- Conversion Procedures:
  After regrouping, student does not properly convert (reduce) the digit regrouped from

Addition Error Pattern:
- Addition Fact:
  Student misses various addition facts by -1 (counting strategy???)
Case Study in Error Analysis

Summarize Error Patterns

- **Facts:**
  - Addition Facts (plus -1)
  - Multi. & Division >5
  - Sign Identification
- **Subtraction Computation:**
  - Conversion procedure
- **Addition Computation:**
  - No major problems
  - Addition Facts (plus 1)

Procedural Errors: Multiplication

7 Separate Steps:

1. **Multiplication Facts:** Are the single digit multiplication processes completed accurately?
2. **Multiplying All Combinations:** Are all different combinations of multiplication attempted?
3. **Regroup (Inside):** Are regroups assigned to the proper column?
4. **Regroup (Outside):** Is the last regrouping part of the product?
5. **Adding the Regroup:** Are regrouped numbers combined with the proper column?
6. **Lining up Addition:** Are the intermediate products lined up correctly?
7. **Addition:** Is the final addition process carried out properly.
Procedural Errors: Division

7 Separate Steps:

1. **Correct Multipliers:** Are the correct multipliers being chosen?
2. **Multiplication:** Regardless of what multipliers are being chosen, is the multiplication being carried out correctly?
3. **Alignment:** Is the result of the multiplication aligned in the correct position?
4. **Subtraction:** Is subtraction completed correctly?
5. **Bringing Down:** Does the student bring down the correct digit at the appropriate times?
6. **Stopping:** Is the procedure stopped at the appropriate time?
7. **Fraction/Decimal:** Are remainders dealt with appropriately?

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Breakout Activity

Identify the Error Patterns

- Identify the error patterns that are present in each student’s solutions.
- Describe the error pattern that is present in very specific and clear terminology.
  - *Can’t subtract is not specific*
  - *Doesn’t know how to divide fractions*
- Reproduce the student’s error on the unsolved problems (FYI---This is a teacher exercise not for kids)
- Discuss reasons why the student might be using the erroneous process.
Reproduce the Error Pattern

A. 354  
   + 541  
   ----  
   895  

B. 63  
   + 91  
   ----  
   154  

C. 324  
   + 489  
   ----  
   813  

D. 467  
   + 732  
   ----  
   1199  

---

Reproduce the Error Pattern

A. 745  
   + 423  
   ----  
   1168  

B. 69  
   + 71  
   ----  
   140  

---

Reproduce the Error Pattern

A. 345  
   - 23  
   ----  
   322  

B. 298  
   - 35  
   ----  
   263  

C. 672  
   - 21  
   ----  
   651  

---

A. 867  
   - 55  
   ----  
   812  

B. 949  
   - 36  
   ----  
   913  

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From Ashlock, 2010
Reproduce the Error Pattern

A. \[ \frac{7}{3} \div 8 = \frac{56}{5} - \frac{24}{5} = \frac{24}{5} - \frac{24}{5} = \frac{48}{5} - \frac{24}{5} = \frac{24}{5} \]

B. \[ \frac{7}{3} \div 6 = \frac{3643}{3} - \frac{42}{3} = \frac{3221}{3} = \frac{31}{3} - \frac{30}{3} = \frac{1}{3} \]

C. \[ \frac{9}{5} \div 7 = \frac{6336}{5} = \frac{36}{5} - \frac{35}{5} = \frac{31}{5} - \frac{30}{5} = \frac{1}{5} \]

D. \[ \frac{9}{4} \div 5 = \frac{4531}{4} = \frac{31}{4} - \frac{30}{4} = \frac{1}{4} \]

A. \[ 3 \boxed{2} 7 \boxed{2} 1 \]

B. \[ 5 \boxed{2} 5 \boxed{4} 5 \]

Reproduce the Error Pattern

A. \[ \frac{1}{5} + \frac{4}{5} = \frac{5}{5} = 1 \]

B. \[ \frac{3}{8} + \frac{1}{4} = \frac{8 \times 1}{3 \times 4} = \frac{8}{12} = \frac{2}{3} \]

C. \[ \frac{3}{4} \div \frac{2}{5} = \frac{4 \times 5}{3 \times 2} = \frac{20}{6} = \frac{10}{3} \]

A. \[ \frac{2}{7} + \frac{1}{3} = \frac{6 + 7}{21} = \frac{13}{21} \]

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Resources for Error Patterns


Correcting Student Errors

*Reteaching ~ Remediation ~ Corrections*

- Critical step to help students overcome their learned error patterns
- Errors will continue if not addressed through targeted and specific instruction.
- Errors do not naturally correct themselves over time
- Practice is a part of correcting errors, but NOT sufficient to alleviate student errors.
- Teachers have many different options
Correcting Student Errors

Reteaching ~ Remediation ~ Corrections

CRA
Small group

Explicit Instruction
Peer Tutoring

Computer guided practice
Instructional Scaffolding

Interleaved Worked Solution Strategy

Micro-Instruction

- Targeting the specific error identified during the error analysis
- Pinpoint instruction
- Teacher directed and controlled instruction and practice
- Focusing the student on the “place” in the process where the error occurred
- Very different than traditional reteaching the whole process
Implications for Instruction

- Use of effective teaching strategies (explicit instructional techniques from previous webinars)
  - Pinpoint error types and efficiently teach or reteach the skill
- Determine the instructional strategy to teach the skill.
- Be sure necessary preskills for strategies are present.
  - Do students understand big idea (i.e., concept)?
  - Do students know facts in a computation problem?

Tyler’s Error Pattern

Conversions in Subtraction with Regrouping

- Remind students of the rule relationship
  - If the bottom number is bigger than top number, you must regroup
  - More place value based language in terms of **subtracting more ones than are available**
- Teach steps of subtraction conversion
  - Subtract from column where regrouping
  - Convert the number from which you regrouped
  - Regroup number in appropriate column
  - Continue Subtraction

Stein, Silbert, & Carnine, (1997)
Instructional Progression

- 3 to 5 examples
  - Teacher directed and then guided practice
  - Teacher support is faded
- Conversions required in different place values in the problem
- Discrimination items (non-example)
  - Examples where conversions are not required
- **Focus instruction on the error pattern and NOT the entire problem process.**

Examples and Non-examples

**Why?**

- Establish the boundaries of rule
- Application of rules
- Under-generalization and overgeneralization
- Focus on the critical attributes(s)
- Textbooks do a poor job with non-examples in teacher demonstration

Archer & Hughes, 2011
### Teaching Subtraction Conversions: Example Problems

<table>
<thead>
<tr>
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</tr>
</thead>
<tbody>
<tr>
<td>-16</td>
<td>-92</td>
</tr>
<tr>
<td></td>
<td>4</td>
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<table>
<thead>
<tr>
<th>97</th>
<th>522</th>
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<tbody>
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### Classroom Implementation

- Teachers bring 3-4 samples of **ONE** student’s work to a grade level planning meeting with a “semi-completed” error analysis.
- Each teacher presents their student’s work and the error analysis with special attention given to the **error pattern**.
- Discuss and develop a **micro instruction** mini lesson progression to implement for this student as well as other students with similar error patterns.
Summary of Error Analysis

Error analysis
- Involves reviewing the student’s independent work (e.g., seatwork, chapter tests, quizzes, progress monitoring probes) to identify specific error types and patterns
- Helps to set priorities for teaching

Error patterns
- Indicate areas in need of further instruction
- Constitute a database for determining what content and strategies to teach

Micro Instructional Remediation
- Specifically Targets are of error
- Focuses attention on specific error versus the entire problem solution
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Example Problems

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